HC49U 4.9152 MHz crystals measurement by circle fit

First, some constants

Marked frequency $f0 := 4.9152 \cdot 10^6$

Scan file name template sfnmt := "HC49U_DRT_4M9152_"

Reference crystal data file name rxdfn := "HC49U_DRT_4M9152_fine.csv"

Min and max crystal number n_min := 0   n_max := 17

Min and max scan number m_min := 0   m_max := 2

Reflection test setup Z0 z0r := 50

Transmission test setup Z0 z0t := 50

Then some functions to start with scans

Num to two-digit string

num2str2d(x) := if(strlen(num2str(x)) < 2, concat("0", num2str(x)), num2str(x))   num2str2d(7) = "07"

Combine file name of scsn type, crystal number and scan number

comb_fn(type, n, m) := concat(sfnmt, num2str2d(n), "_", type, "_", num2str2d(m), ".csv")

comb_fn("avg", 11, 0) = "HC49U_DRT_4M9152_11_avg_00.csv"

Read average (full, both series and parallel resonanses), series only and parallel only resonance scan file of reflection and transmission mode number m of crystal number n

Read a particular CSV scan by name rd_ccv_nm(name) := READCSV(name, 3)

Read a CSV scan by type, n, m rd_scan(type, n, m) := READCSV(comb_fn(type, n, m), 3)

Rho to Z $\mathrm{rh2z(rho)} := \mathrm{z0r} \cdot \frac{1 + \mathrm{rho}}{1 - \mathrm{rho}}$

Freq & Rho to Freq & Z (reflection) $\mathrm{frh2fz(rcsv)} := \mathrm{augment}\left(\mathrm{rcsv}^{\langle 0 \rangle}, \mathrm{rh2z}\left(\mathrm{rcsv}^{\langle 1 \rangle} + j \cdot \mathrm{rcsv}^{\langle 2 \rangle}\right)\right)$

Freq & Z from average reflection scan rd_fz_ar(n, m) := frh2fz(rd_scan("avg", n, m))

Freq & Z from series reflection scan rd_fz_sr(n, m) := frh2fz(rd_scan("ser", n, m))

Freq & Z from parallel reflection scan rd_fz_pr(n, m) := frh2fz(rd_scan("par", n, m))

Read average reflection CSV scan rd_ar(n, m) := rd_scan("avg", n, m)

Read series reflection CSV scan rd_sr(n, m) := rd_scan("ser", n, m)

Read parallel reflection CSV scan rd_pr(n, m) := rd_scan("par", n, m)

Uncalibrated K to Z $\mathrm{unc\_k2z(k)} := 2 \cdot \mathrm{z0t} \cdot \frac{1 - k}{k}$

Uncalibrated Freq & K to Freq & Z (transmission)

$\mathrm{unc\_fk2fz(rcsv)} := \mathrm{augment}\left(\mathrm{rcsv}^{\langle 0 \rangle}, \mathrm{unc\_k2z}\left(\mathrm{rcsv}^{\langle 1 \rangle} + j \cdot \mathrm{rcsv}^{\langle 2 \rangle}\right)\right)$

Uncalibrated Freq & Z from series transmission scan

unc_rd_fz_st(n, m) := unc_fk2fz(rd_scan("res", n, m))

Calibrate transmission factor and offset

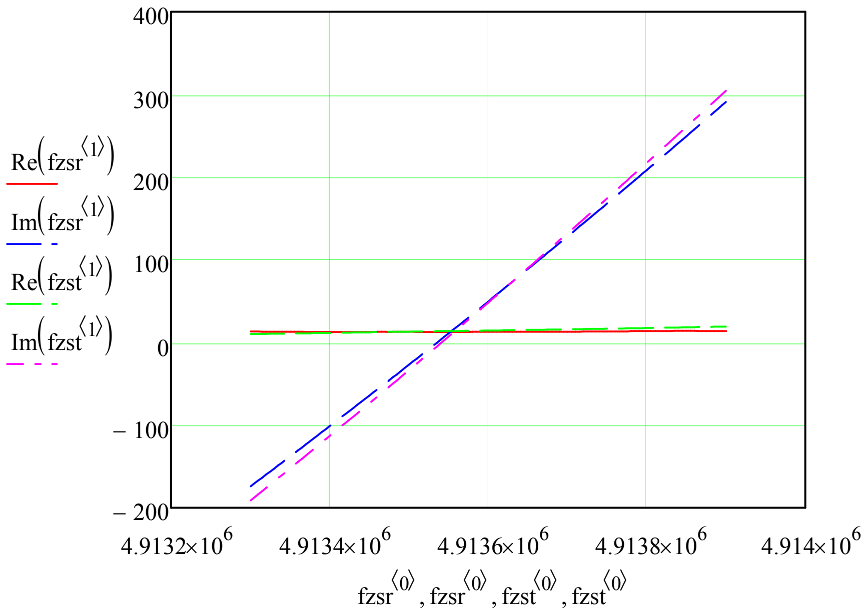
Since my VNA was calibrated only for k=1 in transmission mode, tune it against reflection data now

Take 0-th series resonance scans (transmission and reflection) of 17-th crystall for calculation:

Reflection scan (golden standard) $fzsr := rd\_fz\_sr(17,0)$

Uncalibrated transmission scan $fzst := unc\_rd\_fz\_st(17,0)$

Uncalibrated transmission Z data slightly differs (R+jX plot):



For calibration we use linear fit in two end points

First point index $na := 0$

Second point index $nb := rows(fzsr) - 1$ $nb = 120$

z0t factor $kzt := \left[\left(fzsr^{\langle 1\rangle}\right)_{nb} - \left(fzsr^{\langle 1\rangle}\right)_{na}\right] \div \left[\left(fzst^{\langle 1\rangle}\right)_{nb} - \left(fzst^{\langle 1\rangle}\right)_{na}\right]$

z0t offset $bzt := \left(fzsr^{\langle 1\rangle}\right)_{nb} - kzt\cdot\left(fzst^{\langle 1\rangle}\right)_{nb}$

The result $kzt = 0.938 + 0.016i$ $bzt = 0.694 + 5.083i$

Now we can continue with data access functions:

Calibrated K to Z $k2z(k) := 2\cdot z0t\cdot\frac{1-k}{k}\cdot kzt + bzt$

Freq & K to Freq & Z (transmission) $fk2fz(rcsv) := augment\left(rcsv^{\langle 0\rangle}, k2z\left(rcsv^{\langle 1\rangle} + j\cdot rcsv^{\langle 2\rangle}\right)\right)$

Freq & Z from average transm. scan $rd\_fz\_at(n,m) := fk2fz(rd\_scan("gva",n,m))$

Freq & Z from series transm. scan $rd\_fz\_st(n,m) := fk2fz(rd\_scan("res",n,m))$

Freq & Z from parallel transm. scan $rd\_fz\_pt(n,m) := fk2fz(rd\_scan("rap",n,m))$

Read average transmission CSV scan $rd\_at(n,m) := rd\_scan("gva",n,m)$

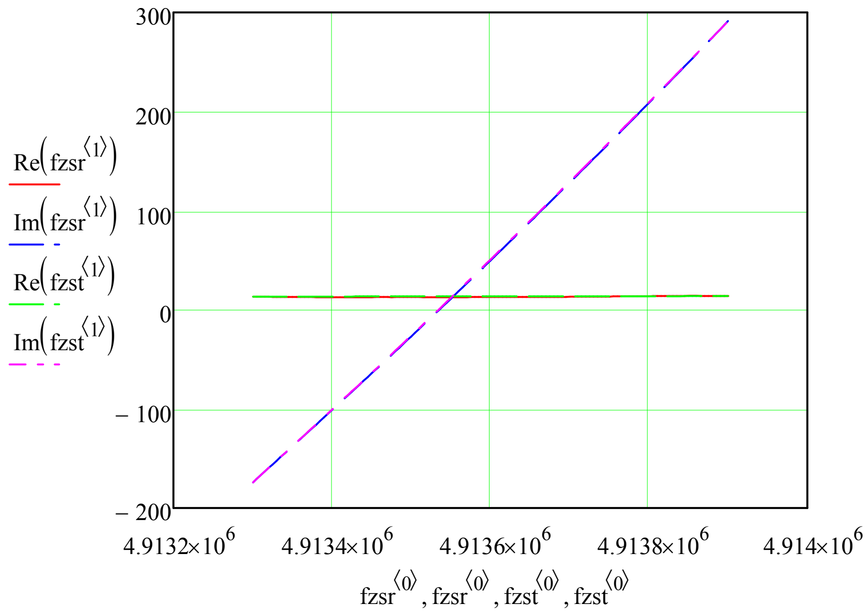
Read series transmission CSV scan $\mathrm{rd\_st}(n,m) := \mathrm{rd\_scan}(\text{"res"}, n, m)$

Read parallel transmission CSV scan $\mathrm{rd\_pt}(n,m) := \mathrm{rd\_scan}(\text{"rap"}, n, m)$

Repeat data comparison to check the functions:

Calibrated transmission scan $\mathrm{fzst} := \mathrm{rd\_fz\_st}(17, 0)$

Calibrated transmission Z data matching is good enough



More functions for scan data processing

Circle fit, as Randy Bullock prescribes

$$\mathrm{cir\_fit}(xv, yv) :=$$

$$n \leftarrow \mathrm{rows}(xv)$$

$$x\_m \leftarrow \frac{1}{n} \cdot \sum_{i=0}^{\mathrm{last}(xv)} xv_i$$

$$y\_m \leftarrow \frac{1}{n} \cdot \sum_{i=0}^{\mathrm{last}(yv)} yv_i$$

$$u \leftarrow xv - x\_m$$

$$v \leftarrow yv - y\_m$$

$$s\_uu \leftarrow \sum_{i=0}^{\mathrm{last}(u)} \left(u_i \cdot u_i\right)$$

$$s\_uv \leftarrow \sum_{i=0}^{\mathrm{last}(u)} \left(u_i \cdot v_i\right)$$

$$s\_vv \leftarrow \sum_{i=0}^{\mathrm{last}(v)} \left(v_i \cdot v_i\right)$$

$$s\_uuu \leftarrow \sum_{i=0}^{\mathrm{last}(u)} \left(u_i \cdot u_i \cdot u_i\right)$$

$$s\_uuv \leftarrow \sum_{i=0}^{\mathrm{last}(u)} \left(u_i \cdot u_i \cdot v_i\right)$$

$$s\_uvv \leftarrow \sum_{i=0}^{\mathrm{last}(u)} \left(u_i \cdot v_i \cdot v_i\right)$$

$$s\_vvv \leftarrow \sum_{i=0}^{\mathrm{last}(u)} \left(v_i \cdot v_i \cdot v_i\right)$$

$$mx \leftarrow \begin{pmatrix} s\_uu & s\_uv \\ s\_uv & s\_vv \end{pmatrix}$$

$$mv \leftarrow \begin{bmatrix} 0.5 \cdot (s\_uuu + s\_uvv) \\ 0.5 \cdot (s\_vvv + s\_uuv) \end{bmatrix}$$

$$uv \leftarrow \mathrm{lsolve}(mx, mv)$$

$$xc \leftarrow uv_0 + x\_m$$

$$yc \leftarrow uv_1 + y\_m$$

$$\alpha \leftarrow \left(uv_0\right)^2 + \left(uv_1\right)^2 + \frac{s\_uu + s\_vv}{\ldots}$$

$$\left|\begin{array}{l} \quad \left(\ \ 0\right) \quad \left(\ \ 1\right) \qquad \text{rows}(u) \\ r \leftarrow \sqrt{\alpha} \\ \begin{pmatrix} xc \\ yc \\ r \end{pmatrix} \end{array}\right.$$

Test circle fit on Randy's example

$$\text{cir\_fit}\left[\begin{pmatrix} 0 \\ 0.5 \\ 1 \\ 1.5 \\ 2 \\ 2.5 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0.25 \\ 1 \\ 2.25 \\ 4 \\ 6.25 \\ 9 \end{pmatrix}\right] = \begin{pmatrix} -11.839 \\ 8.446 \\ 14.686 \end{pmatrix}$$

Some functions for geting index of the first element in vector beyond n-th for a given condition

First inside the range

$$\text{fst\_inr}(v, n, \text{min}, \text{max}) := \left|\begin{array}{l} i \leftarrow n \\ \text{for}\ \ i \in (n .. \text{last}(v)) \\ \quad \left|\begin{array}{l} \text{continue} \quad \text{if}\ \ v_i < \text{min} \\ \text{continue} \quad \text{if}\ \ v_i > \text{max} \\ \text{break} \end{array}\right. \\ i \end{array}\right.$$

First outside the range

$$\text{fst\_otr}(v, n, \text{min}, \text{max}) := \left|\begin{array}{l} i \leftarrow n \\ \text{for}\ \ i \in (n .. \text{last}(v)) \\ \quad \left|\begin{array}{l} \text{break} \quad \text{if}\ \ v_i < \text{min} \\ \text{break} \quad \text{if}\ \ v_i > \text{max} \end{array}\right. \\ i \end{array}\right.$$

Get the range start and stop

$$\text{get\_rng}(v, n, \text{min}, \text{max}) := \left|\begin{array}{l} i \leftarrow \text{fst\_inr}(v, n, \text{min}, \text{max}) \\ j \leftarrow \text{fst\_otr}(v, i, \text{min}, \text{max}) \\ \begin{pmatrix} i \\ j - 1 \end{pmatrix} \end{array}\right.$$

Have a look at series resonance scan of one crystall

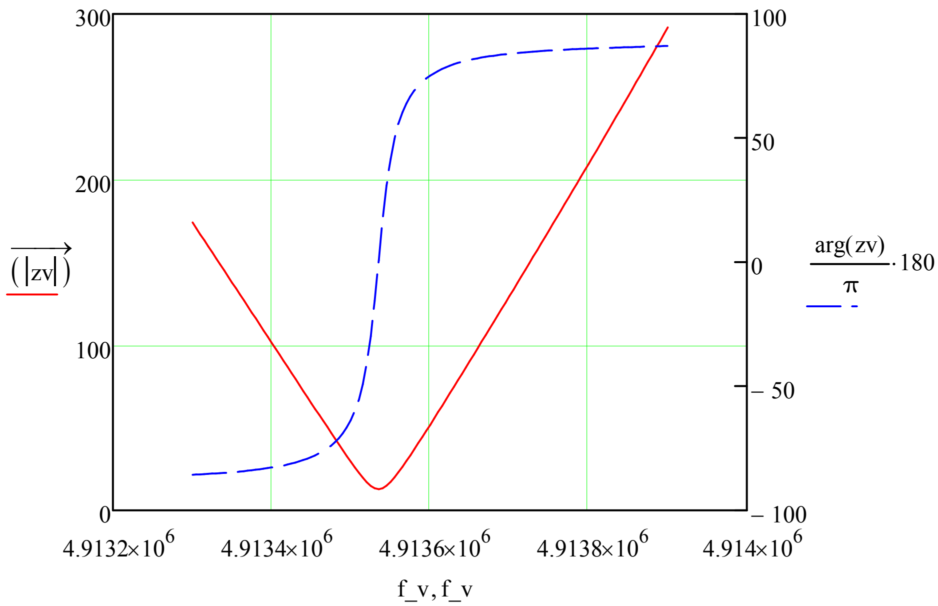
Crystall index, scan index $\quad \text{xqn} := 17 \qquad \text{scn} := 0$

Read reflection scan $\quad \text{fzsr} := \text{rd\_fz\_sr}(\text{xqn}, \text{scn})$

Frequency vector $\quad \text{f\_v} := \text{fzsr}^{\langle 0 \rangle}$ (fv name is used by Mathcad function)

Impedance vector $\quad \text{zv} := \text{fzsr}^{\langle 1 \rangle}$

Classic Z(f) view in mod/arg form:



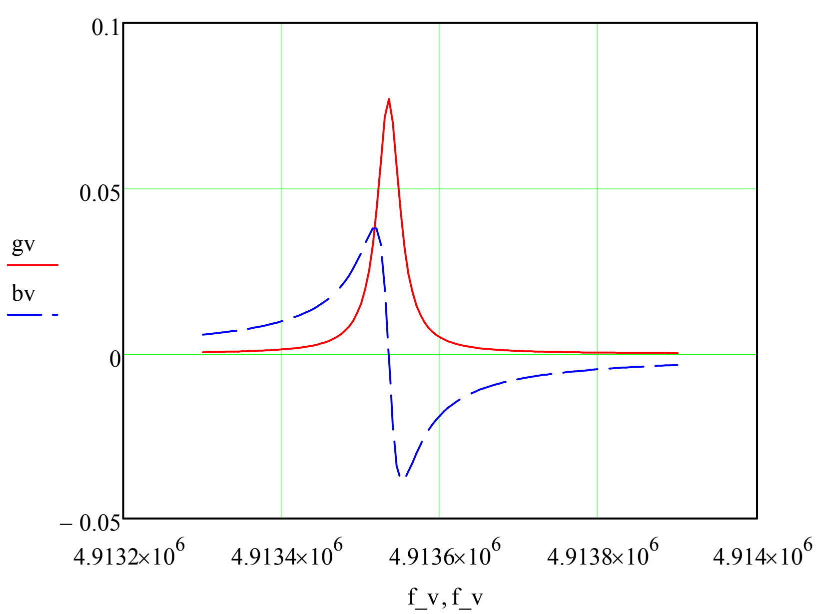
Admittance vector $$yv := \frac{1}{zv}$$

For circle fitting split the admittance:

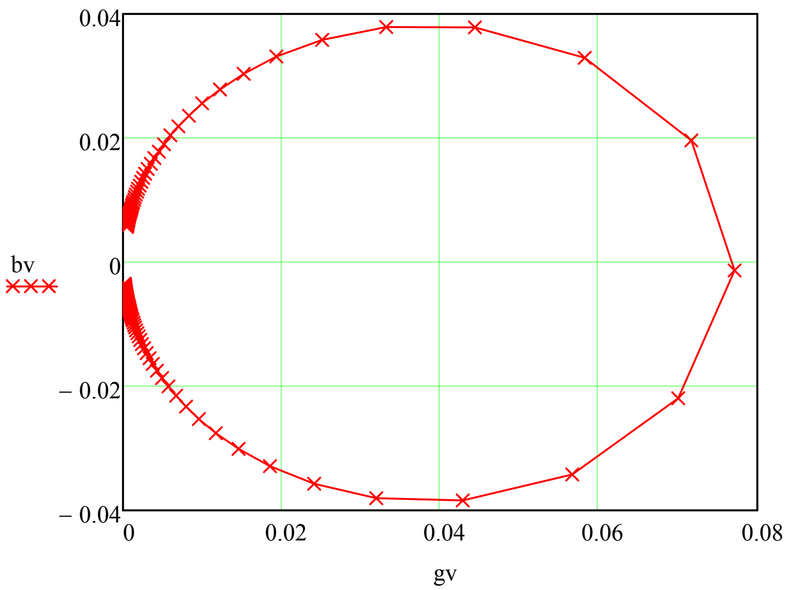
Conductance $$gv := \mathrm{Re}(yv)$$

Susceptance $$bv := \mathrm{Im}(yv)$$

Due to duality, G+jB view of series resonance is similar to R+jX plot of a parallel one



But what we are interested in is its B/G plot:



And now it's time for circle fitting: $\begin{pmatrix} g0 \\ b0 \\ gr \end{pmatrix} := \mathrm{cir\_fit}(gv, bv)$

(g0,b0) - centre, gr - radius $g0 = 0.038$ $b0 = -4.614 \times 10^{-5}$ $gr = 0.038$

Series resistance based on the centre $Rs\_c := 0.5 \cdot g0^{-1}$ $Rs\_c = 12.988$

Based on the radius, good match $Rs\_r := 0.5 \cdot gr^{-1}$ $Rs\_r = 12.999$

To compare against your Rm value for this crystal the CSV file with measured parameters was manually edited: commas at the end of each line confused Mathcad. Good matching IMHO:

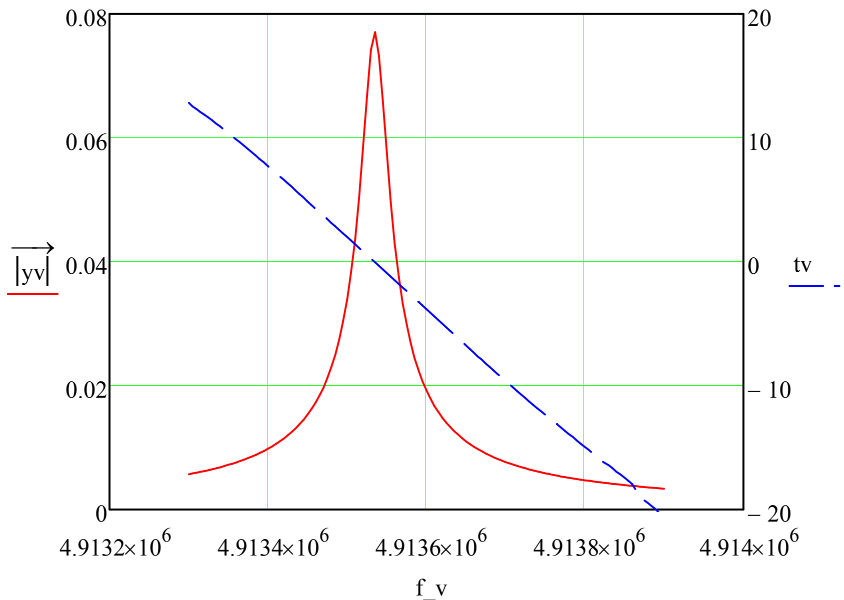
$$msd\_dat := \mathrm{READCSV}\left[\mathrm{concat}(rxdfn, ".txt"), \begin{pmatrix} 2 \\ 2 + n\_max \end{pmatrix}\right] \qquad msd\_dat_{xqn, 5} = 12.8$$

It seems possible to calculate RMS of radius deviation for all points against the fitted circle, it can be used as tolerance estimation for the resulting Rm parameter. Unfortunately b0 is too far from expected C0 susceptance:

C0 calculated on b0 $c0a := \dfrac{b0}{2 \cdot \pi \cdot f0}$ $c0a = -1.494 \times 10^{-12}$

The next stage of curve fitting is much simpler - it's just a line fit into tan(phase) near resonance

Tangent of admittance phase $tv := \tan(\arg(yv))$ (tan(arg(zv)) has only flipped sign)



What I saw on my crystals was ideal line from some -15 to +15 of tangent value

Select subvector to be used for fitting $\begin{pmatrix} nx1 \\ nx2 \end{pmatrix} := \text{get\_rng}(tv, 0, -10, 10)$

Start index $nx1 = 12$ $\quad f\_v_{nx1} = 4.91336 \times 10^6$

Stop index $nx2 = 81$ $\quad f\_v_{nx2} = 4.913705 \times 10^6$ 5Hz step

Take subvector of frequency values $lfv := \text{submatrix}(f\_v, nx1, nx2, 0, 0)$

And tangent values $ltv := \text{submatrix}(tv, nx1, nx2, 0, 0)$

Fit the line; Mathcad complains on some non-reality in lfv (!?), so Re $\begin{pmatrix} a0 \\ a1 \end{pmatrix} := \text{line}(\text{Re}(lfv), ltv)$

Line slope gives Q $Q := 0.5 \cdot |a1| \cdot f0$ $\quad Q = 1.421 \times 10^5$

Zero phase frequency $zpf := -\frac{a0}{a1}$ $\quad zpf = 4.913534 \times 10^6$

Motion inductance $Lm := \frac{Q \cdot Rs\_r}{2 \cdot \pi \cdot f0}$ $\quad Lm = 0.059821$

Compare to your results, Q $msd\_dat_{xqn, 8} = 1.451 \times 10^5$

Zero phase frequency $msd\_dat_{xqn, 1} = 4.913537 \times 10^6$

Motion inductance $msd\_dat_{xqn, 6} = 0.060144$

After calculating C0 using a parallel resonance frequency (high accuracy isn't needed there) it is possible to take it into account for phase in series RLC branch and calculate the true series resonance frequency.

The circle fitting technique is described in the 4,782,281 US patent (test system is claimed, http://patft.uspto.gov/netacgi/nph-Parser?Sect1=PTO1&Sect2=HITOFF&p=1&u=/netahtml/PTO/src

hnum.html&r=1&f=G&l=50&d=PALL&s1=4782281.PN.), however it seems completely legal to use it. The best description of the circle fitting algorithm is probably by Randy Bullock, https://dtcenter.org/met/users/docs/write_ups/circle_fit.pdf