Designing the Z90's Gaussian Crystal Filter

This article presents a detailed design review of the Gaussian crystal filters in the Z90 panadapter.

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A critical part of the Z90 panadapter described in the March/April 2007 issue of *QEX* are the two crystal filters. Because a panadapter's filter has different requirements than a receiver filter for CW or SSB, and because filter design is often — but incorrectly — viewed as "black magic" we'll look at the filter in more detail. Although centered around a Gaussian filter, the procedure is identical for other filter types.

Before settling on the design discussed in this article, I built more than a dozen experimental crystal filters, with center frequencies from 3.58 MHz to 13.5 MHz. Based on that work, my design employs inexpensive microprocessor crystals manufactured by ECS, Inc International (**www.ecsxtal.com**). I used part number ECS-80-S-1X 8.0, with a nominal 8 MHz center frequency.[1] We start the design by selecting the target bandwidth, filter response type and number of sections:

Bandwidth

The target bandwidth is a judgment call. Our design permits two filters, "narrow" and "wide." Commercial analog spectrum analyzers provide automatic bandwidth selection between about 0.3% and 2% of the total frequency span. To analyze an SSB signal — a common use of a panadapter —a total span of about 10 kHz is appropriate. This suggests a bandwidth of perhaps 30 Hz to 200 Hz for the narrow filter. Experience with inexpensive microprocessor crystals shows that at 8 MHz bandwidths below 200 Hz become increasingly difficult due to crystal *Q* limitations, so we set the target narrow bandwidth at 200 Hz. Further, the narrower the filter, the slower the sweep speed must be to avoid amplitude and frequency errors.[2]

Also entering into the bandwidth choice is the Z90's maximum horizontal sample size of 240 data points. To reduce amplitude-measuring errors, the filter should have a 3 dB bandwidth significantly greater than the data point spacing. With a 20 kHz span, the data points are spaced 83.3 Hz apart. With a 200 kHz span, representing viewing ±100 kHz from the tuned frequency, data points are spaced every 833 Hz. A 1-kHz-bandwidth "wide" filter is a reasonable choice, albeit a bit on the narrow side. Perhaps more importantly, experience shows that as the filter bandwidth increases much beyond 1 kHz for these 8 MHz crystals, our simple circuit topology fails to provide acceptable symmetry. Hence, we select 1 kHz as a reasonable "wide" bandwidth.

Filter Response Type

By juggling passband flatness against how fast the passband rolls off and other parameters, designers have developed many filter types, such as Butterworth, Chebyshev, Bessel and Gaussian. Gaussian or near Gaussian filters are traditionally in a spectrum analyzer or panadapter.[3, 4] A Gaussian filter provides an optimum amplitude response as it is swept through continuous or pulsed waveforms. (A true Gaussian filter requires an infinite number of elements; our four-section filter is, however, "close enough" to Gaussian response to be more than acceptable.)

Number of Sections

Increasing the number of filter sections improves the flank selectivity, or rejection of signals far outside the passband. Increasing the number of filter sections, however, makes the filter more demanding in terms of component quality and tolerance. As a judgment call, the Z90 uses a four-section filter. The standard reference on filter design is Zverev's *Handbook of Filter Synthesis*.[5] That book shows that a four-section Gaussian filter has a –60 dB bandwidth that is 10 times the –3 dB bandwidth, so our filter will have a 3 dB: 60 dB shape factor of 10:1. (Spectrum analyzers traditionally use a 3 dB: 60 dB shape factor, instead of the 6 dB:60 dB shape often found in Amateur Radio specifications.) Even if we use a 10-section Gaussian filter, the shape factor only improves to 4.8:1.

Characterize the Crystal Resonators to be Used

It is impossible to approach filter design in a systematic fashion without a decent knowledge of the motional parameters of the crystals to be used. As James Clerk Maxwell, of Maxwell equation fame, observed some 130 years ago:

"The most important aspect of any phenomenon from a mathematical point of view is that of a measurable quantity."[6]

Lord Kelvin expressed similar views:

"When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the state of science."[7]

Figure 1 shows the simplest useful

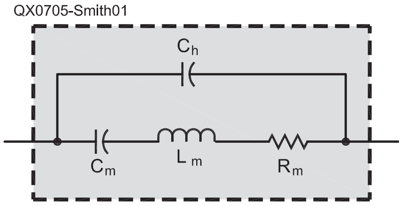

Figure 1 — Simple but useful model of a quartz crystal.

[1]Notes appear on page 26.

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electrical model of a crystal.[8, 9, 10] It has four elements:

C_h — the capacitance of the holder, including the plated electrodes on either side of the quartz blank

C_m — the *motional* capacitance

L_m — the *motional* inductance

R_m — the *motional* resistance

The parameters C_m, L_m and R_m are called "motional" elements because they are electrical analogs of the physical vibrations of the quartz element; in other words, they result from motion of the quartz plate, with L_m corresponding to the plate's mass, C_m corresponding to its elasticity and R_m to frictional loss. (In the professional literature, C_h is identified as C_0, and the motional parameters as C_1, L_1 and R_1.) Nonetheless, C_m, L_m and R_m may be measured and are as real to us as any physical capacitor, inductor or resistor. Removing the protective case from a crystal, of course, does not reveal three tiny circuit components. Rather, you will find a thin circular quartz blank with electrodes deposited onto both sides.

The parameters shown in Figure 1 may be measured through a variety of methods. We will compare three:

1. Derive the parameters by measuring the holder capacitance and the series and parallel resonant frequencies of the crystal;
2. Use the oscillator-frequency-shift method developed by G3UUR;
3. Compare these two measurements with data from an HP87510A gain-phase analyzer's automatic crystal resonator characterization.

Holder Capacitance

Before measuring the motional parameters, we first measure the holder capacitance. Although we won't use C_h in our narrow band filter design equations, it is an important parameter in more sophisticated designs, particularly for wider bandwidth filters, as described in Dishal's classic paper.[11, 12] We'll also see C_h used in some methodologies to calculate motional parameters. Hence, it is important to measure C_h as accurately as we can. To disentangle it from the motional parameters, C_h should be measured at a frequency far below the crystal's resonant frequency. Table 1 summarizes my holder capacitance measurements. Disregarding the one 4.13 pF value as an outlier, the holder capacitance can be taken as the average of the remaining five measurements, 3.83 pF.

Series and Parallel Resonance Method

If we place the crystal to be measured in series between a signal generator and a signal level detector, as we tune the generator near the frequency marked on the crystal can, we see two points of resonance. The first produces a transmission maximum and corresponds to the series resonant frequency, or the frequency at which C_m and L_m resonate. If we tune the signal generator a bit higher in frequency, however, we see a transmission minimum, corresponding to parallel resonance of C_h with the series combination of C_m and L_m. If our equipment permits, we observe a phase shift as well — at both the series and parallel resonant frequen-

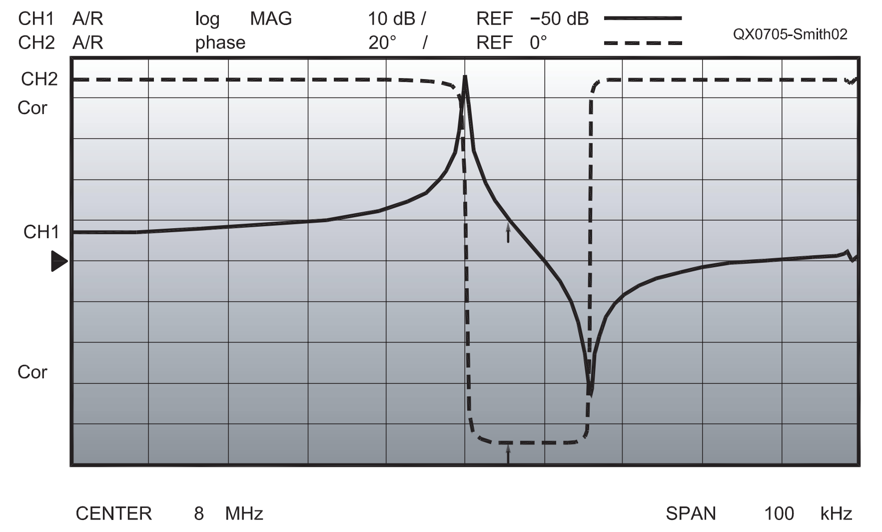

Figure 2 — Impedance and phase of a crystal at both series and parallel resonance.

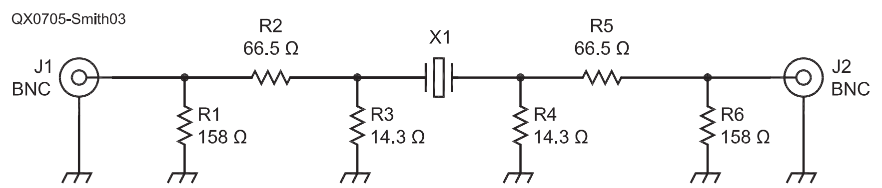

Figure 3 — Resistive pi fixture for measuring crystal parameters.

Table 1
Crystal Holder Capacitance Measurements

Method	*Value*	*Comments*
Digital capacitance meter	3.7 pF	BP Model DCM-601 meter.
Resolution	0.1 pF	
Gain-Phase Analyzer		
HP87510A	4.13 pF	@ 100 kHz
	3.89 pF	@ 300 kHz
	3.83 pF	@ 1 MHz
	3.90 pF	@ 2 MHz
HP4342A Q-meter with Boonton 103A-32 2.5 mH working coil via resonance shift method	3.85 pF	@ 145 kHz
Mean	3.83 pF	Exclude 4.13 pF data point

cies the phase shift is zero.[13] Figure 2 shows a transmission amplitude and phase plot for the 8 MHz crystals used in our filter design.

The measurements in this article are taken with an HP87510A gain-phase meter, a 100 kHz to 300 MHz vector network analyzer, optimized for component measurement, including a feature to automatically compute and display the equivalent parameters of a crystal resonator. The HP87510A combines the function of signal generator and signal strength (and phase) detector. The principles we use, however, are applicable to a variety of measurement equipment. See, Hayward[14] and Pivnichny[15] for test methods not requiring sophisticated equipment.

As convenient as using a network analyzer is, it's not as simple as plugging a crystal into a port on the 87510A. Rather, the crystal must be installed in a test fixture so that it is presented with the correct impedance and drive level, and, for some measurements but not for our purposes, the correct shunt capacitance. Although I do not have the recommended HP41900A crystal holder, it is possible to make a substitute capable of the accuracy we need for filter design. (We consider stray shunting capacitance in the substitute fixture only to the extent it increases C_h, an omission that limits the achievable accuracy.)

The IEC standard for crystal measurement requires the crystal be driven from a 12.5 Ω source and terminated in a 12.5 Ω load.[16] This is traditionally accomplished through back-to-back resistive pi attenuators, providing impedance matching to 50 Ω test equipment, resulting in a net 29.6 dB loss. See Salt[17] or Pivnichny.[18] My version of the attenuator, using standard 1% resistors, is shown at Figure 3 and pictured at Figure 4. Figure 5 shows how the test fixture is connected to the 87510A. 10 dB attenuators (Mini-Circuits Laboratories model CAT-10) at the input and output ports of the fixture further improve the 12.5 Ω impedance match. (Some of my data was taken with an earlier design — a transformer-based 50:12.5 Ω matching test fixture. It exhibited greater error than the resistive fixture.)

It's also important to drive the crystal under test with a signal level representative of the circuit in which it will be used, since motional parameters vary with drive level, a phenomenon known in the industry as "drive level dependency" or DLD. (It's also possible to shatter a crystal with grossly excessive drive.) All test data presented is taken with a signal level of –20 dBm presented to the input of the test fixture. Figures 6 and 7, for example, show R_m varies significantly as the drive level varies from –50 dBm to 0 dBm. (The 10 dB pads were removed for these tests.) Both crystals exhibit abrupt jumps in R_m and show a general trend toward lower R_m with increasing drive. To test whether the jumps were an instrument or calibration artifact, I substituted a 9.1 Ω, ¼ W carbon film resistor for the crystal swept

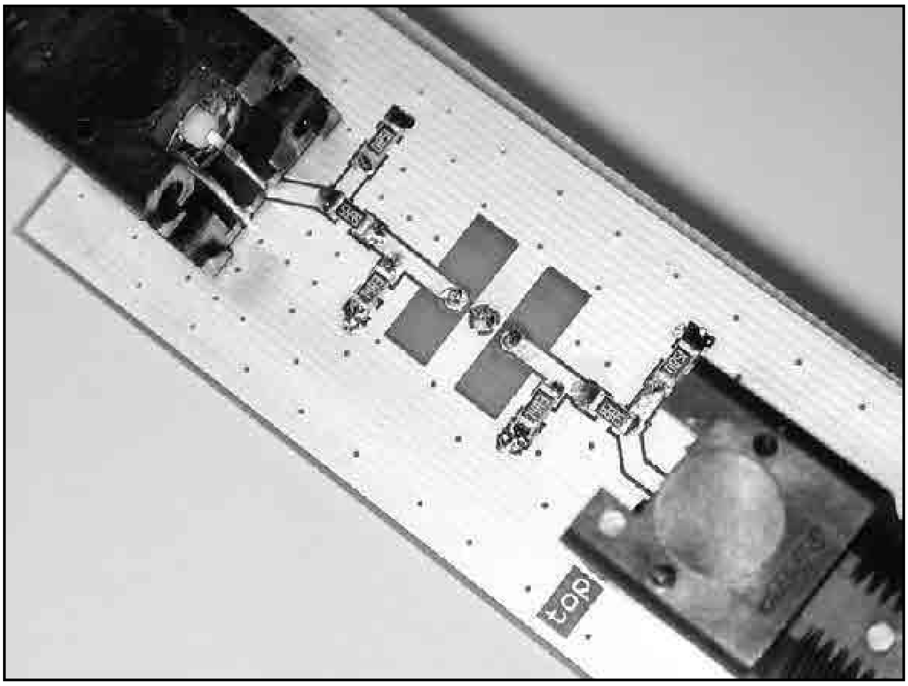

Figure 4 — Resistive pi fixture printed circuit board.

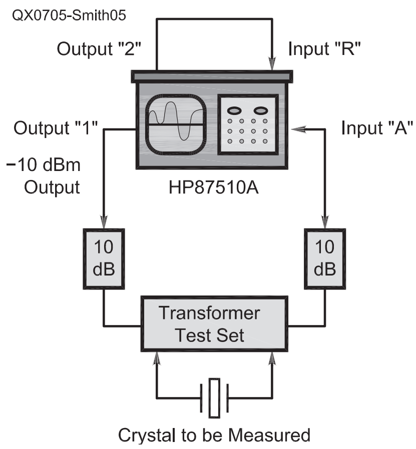

Figure 5 — Setup for measuring crystal parameters using HP87510A VNA.

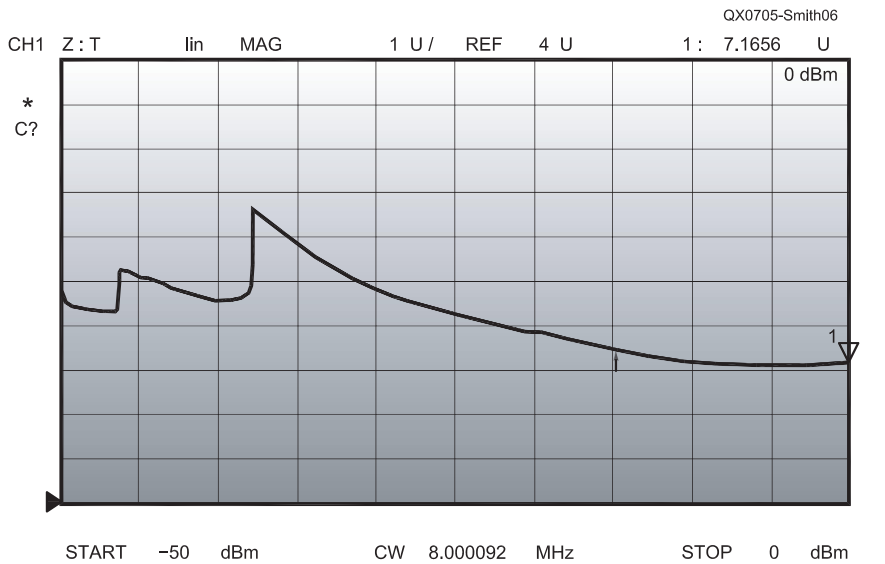

Figure 6 — Variation in crystal series resistance versus drive level. Note the abrupt jumps in resistance as drive is varied.

in Figure 6. As Figure 8 shows, its resistance is essentially unchanged over the –50 to 0 dBm level. Salt describes R_m jumps, similar to those seen in Figures 6 and 7, as resulting from coupling among multiple resonance modes, as the dominant resonance shifts from one mode to another with slight changes in operating conditions.[19] Fortunately for us, these resonance modes differ only slightly in frequency. C_m and L_m also change their values slightly with varying drive levels. (Changes in motional values with applied signal level are why a crystal filter can be a source of intermodulation.)

With the crystal installed in an impedance matching test fixture, we measure four parameters, and, if your setup permits, a fifth parameter:

- Series resonant frequency — f_s
- Parallel resonant frequency — f_p
- Series resonant 3 dB bandwidth — Δf
- Parasitic capacitance of the test fixture — C_p
- (Optional) Series resonant attenuation (dB) — α

Figures 9 and 10 show the swept frequency data for a typical ECS-80-S-1X 8.0 crystal, from which we obtain the parameters summarized in Table 2. The value for C_p was measured separately.

The following motional parameter analysis is based upon Omicron Lab's Application Note.[20] From standard circuit theory, we first state the series and parallel resonant frequencies and the resonator Q in terms of the crystal parameters:

$$f_s = \frac{1}{2\pi\sqrt{L_m C_m}} \quad \text{[Eq 1]}$$

$$f_p = f_s\sqrt{1+\frac{C_m}{C_h}} \approx f_s\left(1+\frac{C_m}{2C_h}\right) \quad \text{[Eq 2]}$$

$$Q = \frac{2\pi\, f_s L_m}{R_m} = \frac{1}{2\pi\, f_s C_m R_m} \quad \text{[Eq 3]}$$

Since these parameters are measured with the crystal installed in the test fixture, we must add the fixture capacitance to the true holder capacitance to find the adjusted holder capacitance, $C_{h\,adj}$:

$$C_{h\,adj} = C_h + C_p = 3.83 \text{ pF} + 0.67 \text{ pF} = 4.50 \text{ pF} \quad \text{[Eq 4]}$$

We recast these equations to solve for the motional parameters in terms of our known parameters f_s, f_p, C_h and Q_L:

$$C_m = 2C_{h\ adj}\left(\frac{f_p}{f_s}-1\right) \quad \text{[Eq 5]}$$

$$C_m = \left(\frac{8.016250 \text{ MHz}}{8.0000625 \text{ MHz}}-1\right)\times 2\times 4.50\text{pF}$$

$$C_m = 0.01821 \text{ pF}$$

$$L_m = \frac{1}{4\pi^2 f_s^2 C_m} \quad \text{[Eq 6]}$$

$$L_m = \frac{1}{4\pi^2 (8.0000625\times 10^6 \text{ Hz})^2 \times 18.21\times 10^{-15} \text{ F}}$$

$$L_m = 21.728 \text{ mH}$$

We next calculate Q_L, the loaded Q of the crystal:

$$Q_L = \frac{f_s}{\Delta f} \quad \text{[Eq 7]}$$

$$Q_L = \frac{8.000\times 10^6 \text{ Hz}}{256.2\text{Hz}} = 31,226$$

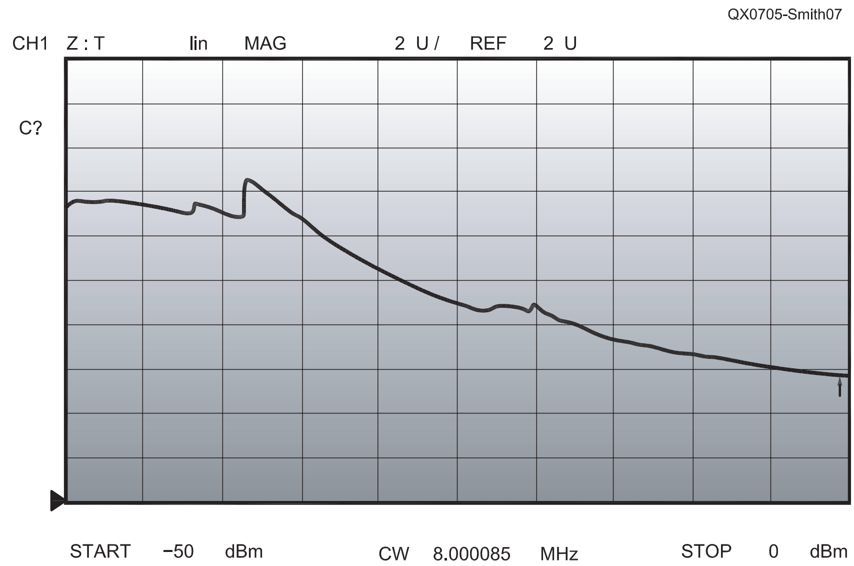

Figure 7 — Second sample crystal shows similar drive level dependency.

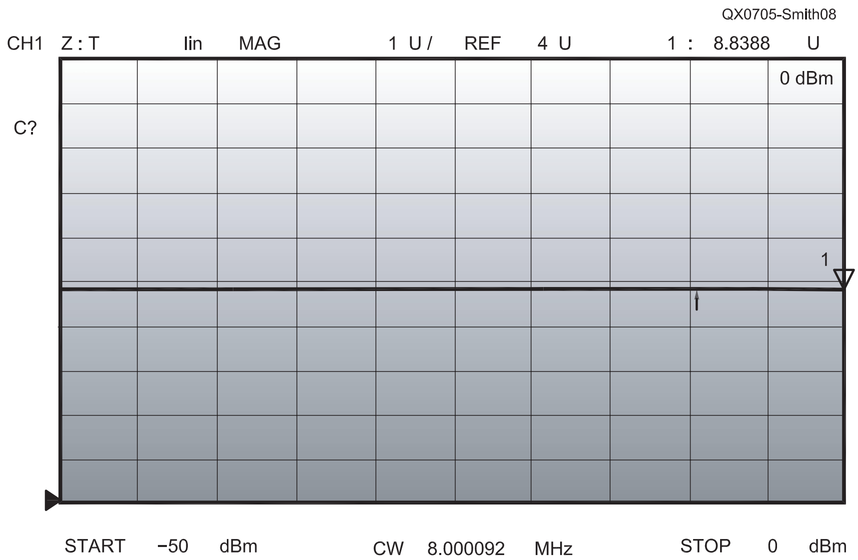

Figure 8 — A resistor shows no indication of drive level dependency.

Table 2
Measured ECS-80-S-1X 8.0 Crystal Parameters

Parameter	*Value*
f_s	8.0000625 MHz
f_p	8.0162500 MHz
Δf	256.2 Hz
C_p	0.67 pF
α	3.1 dB

From the definition of Q, we calculate the series resistance, R_{Total}, seen by the crystal, consisting of R_m plus the source and load resistance. See Figure 11.

$$R_{Total} = \frac{2\pi\ f_s L_m}{Q_L}$$

Solving for R_{Total}:

$$R_{Total} = \frac{2\pi\ f_s L_m}{Q_L}$$

$$R_{Total} = \frac{2 \times \pi \times 8.000 \times 10^6\ \text{Hz} \times 21.728 \times 10^{-3}\ \text{H}}{31226}$$

$$R_{Total} = 34.98\ \Omega$$

The source and load resistances presented to the crystal are both 12.5 Ω, which we subtract from R_{Total} to obtain R_m:

$$R_m = R_{Total} - R_{Termination} \qquad \text{[Eq 8]}$$

$$R_m = 34.98\ \Omega - 12.5\ \Omega - 12.5\ \Omega = 9.98\ \Omega$$

Is it possible to determine the motional values without knowledge of the holder and test fixture capacitance? The answer is yes.

Figure 10 shows the measured attenuation $\alpha = 3.0397$ dB. Assuming this value is correct — and we should certainly not accept attenuation stated to four decimal places as accurate to all four places[21] — we can compute the loaded Q, deriving L_m from it and the source and load impedance:

The relationship between series resistance and attenuation is:[22]

$$R_s = 2R_0\left(10^{\frac{\alpha}{20}} - 1\right) \qquad \text{[Eq 9]}$$

Where:

R_s is the series resistance

R_0 is the impedance of the source and load, assumed equal

α is the attenuation, in decibels

Applying this to our measured data, we determine the motional resistance:

$$R_m = 2 \times 12.5\ \Omega \times \left(10^{\frac{3.0397}{20}} - 1\right) = 10.475\ \Omega$$

The total resistance seen by L_m is thus 10.475 Ω plus the 12.5 Ω source and terminating impedance, or 35.475 Ω total.

Table 3
ECS-80-S-1X 8.0 Crystal Frequencies Measured With G3URR Method

Condition	*Value*
Switch closed, no extra series C	8.000613 MHz
Extra series C	8.002418 MHz
Shift in frequency	1805 Hz

We now find L_m:

$$L_m = \frac{Q \times R_{Total}}{2\pi\ f_s}$$

$$L_m = \frac{31226 \times 35.475\ \Omega}{2 \times \pi \times 8.0000625 \times 10^6\ \text{Hz}}$$

$L_m = 22.03$ mH

C_m can be determined from the series resonant frequency:

$$C_m = \frac{1}{4\pi^2\ f_s^2 L_m}$$

$$C_m = \frac{1}{4 \times \pi \times \left(8.000 \times 10^6\ \text{Hz}\right)^2 \times 22.03 \times 10^{-3}\ \text{H}}$$

$$C_m = 0.01797\ \text{pF}$$

In truth, these calculations trade lack of knowledge about the holder capacitance for lack of knowledge of the exact attenuation and the source and load terminations. We should, however, — at least in principle — be able to measure attenuation and the source and load terminations at least as accurately as the holder and fixture capacitance.

G3UUR Method

David Gordon-Smith, G3UUR, has developed a simple method of measuring crystal parameters, popularized by Wes Hayward.[23] G3UUR's method involves measuring the shift in frequency of a series-tuned Colpitts oscillator when a small capacitor is added in series with the crystal. L_m and C_m are then calculated from the following equations:[24]

$$C_m \approx \frac{2\Delta f}{f}\left(C_h + C_s\right) \qquad \text{[Eq 10]}$$

Where:

C_s is the series capacitor

Δf is the shift in frequency when C_s is inserted.

L_m is determined from the resonant frequency, using Equation 6:

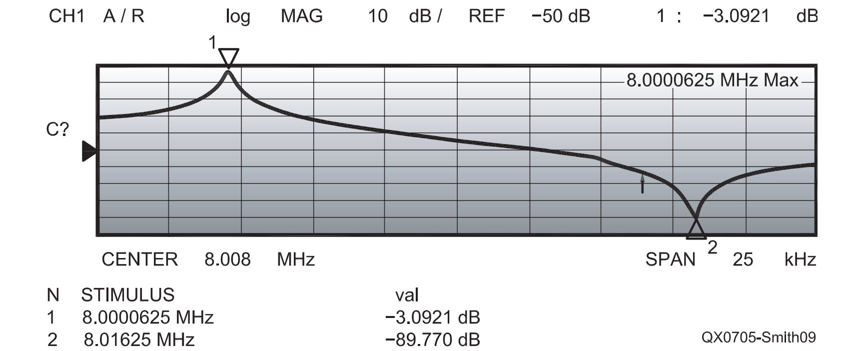

Figure 9 — Swept frequency data for a typical 8 MHz crystal, showing both series and parallel resonant points.

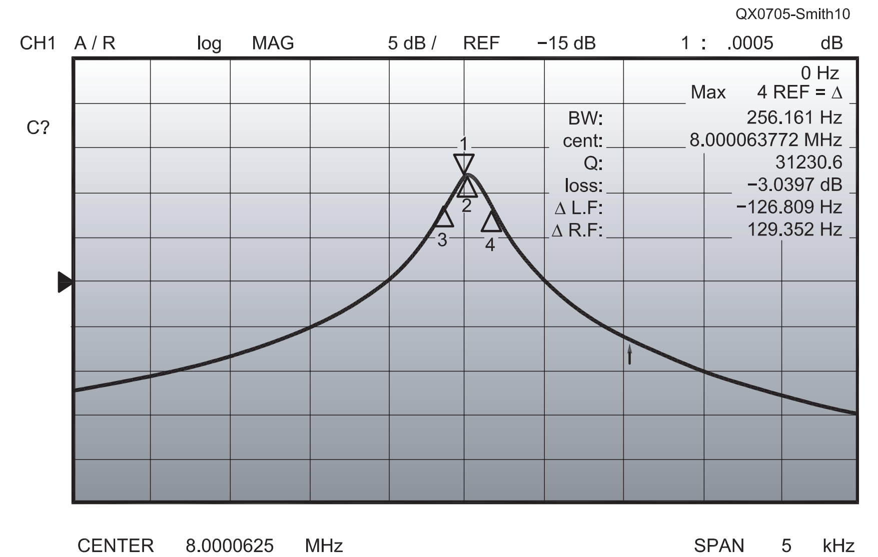

Figure 10 — Expanded view of series resonance in the 8 MHz crystal.